

SM3 6.1: Solving Square Root Equations

Today, we explore solving equations that contain square root expressions, like \sqrt{x} , $\sqrt{3x - 5}$, or $\sqrt{x + 2}$.

In an equation that contains just one root, our plan is to isolate the root, remove the root by squaring, and then continue solving using techniques from previous units. Unfortunately, because the domain of square root functions is not all real numbers, we need to check for extraneous solutions. In the event that we find an extraneous solution, we reject that particular solution.

$\begin{aligned} \sqrt{x - 3} + 7 &= 12 \\ \sqrt{x - 3} &= 5 \\ x - 3 &= 25 \\ x &= 28 \end{aligned}$	<p style="text-align: center;">Solve: $\sqrt{x - 3} + 7 = 12$</p> <p style="text-align: center;">Given</p> <p style="text-align: center;">Subtract 7 to isolate the root</p> <p style="text-align: center;">Square both sides to remove the root</p> <p style="text-align: center;">Continue solving the equation</p>
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Check:

$$\begin{aligned} \sqrt{28 - 3} + 7 &= 12 \\ \sqrt{25} + 7 &= 12 \\ 5 + 7 &= 12 \\ 12 &= 12 \end{aligned}$$

We'll keep our answer of $x = 28$.

In equations that contain two roots, we may not be able to isolate them both. In that case, we isolate just one of them, remove that root by squaring, continue with a problem that may still contain a root, isolate the remaining root, remove the remaining root by squaring, and then continue solving using techniques from previous units. Also, the joy is having to check for extraneous solutions is unabated.

$\begin{aligned} \sqrt{x + 3} &= \sqrt{x + 10} - 1 \\ x + 3 &= (\sqrt{x + 10} - 1)(\sqrt{x + 10} - 1) \\ x + 3 &= x + 10 - \sqrt{x + 10} - \sqrt{x + 10} + 1 \\ x + 3 &= x + 11 - 2\sqrt{x + 10} \\ -8 &= -2\sqrt{x + 10} \\ 4 &= \sqrt{x + 10} \\ 16 &= x + 10 \\ 6 &= x \end{aligned}$	<p style="text-align: center;">Solve: $\sqrt{x + 3} = \sqrt{x + 10} - 1$</p> <p style="text-align: center;">Given</p> <p style="text-align: center;">Square both sides to remove the left root</p> <p style="text-align: center;">Distribute the right side</p> <p style="text-align: center;">Combine like terms</p> <p style="text-align: center;">Subtract x and 3 to isolate the remaining root</p> <p style="text-align: center;">Divide by -2 to isolate the remaining root</p> <p style="text-align: center;">Square both sides to remove the root</p> <p style="text-align: center;">Continue solving the equation</p>
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Check:

$$\begin{aligned} \sqrt{6 + 3} &= \sqrt{6 + 10} - 1 \\ \sqrt{9} &= \sqrt{16} - 1 \\ 3 &= 4 - 1 \\ 3 &= 3 \end{aligned}$$

We'll keep our answer of $x = 6$.

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SM3 6.1

Solve each equation for x .

1) $\sqrt{x} = 5$

2) $\sqrt{x} = 9$

3) $\sqrt{x} = -3$

4) $\sqrt{3x} = 6$

5) $\sqrt{4x} = 10$

6) $\sqrt{2x} = -6$

7) $2\sqrt{x} = 1$

8) $2\sqrt{5x} = 20$

9) $\sqrt{2x - 1} = 7$

10) $\sqrt{3x + 7} - 7 = 0$

11) $\sqrt{x^2 + 16} = x + 2$

12) $\sqrt{x^2 + 5x + 10} = 2$

$$13) \quad \sqrt{4x-3} - \sqrt{x+6} = 0$$

$$14) \quad 2x - \sqrt{16x-12} = 0$$

$$15) \quad 2\sqrt{x+6} - \sqrt{-8x} = 0$$

$$16) \quad \sqrt{x+19} = \sqrt{x+10} + 1$$

$$17) \quad \sqrt{7x+1} = \sqrt{12x+4} - 2$$

$$18) \quad \sqrt{5x+6} + 3 = \sqrt{3x+3} + 4$$

$$19) \quad x - 3 = \sqrt{10x - 54}$$

$$20) \quad x + 1 = \sqrt{7x + 15}$$